High-Performance Modular Finite Element System HERMES

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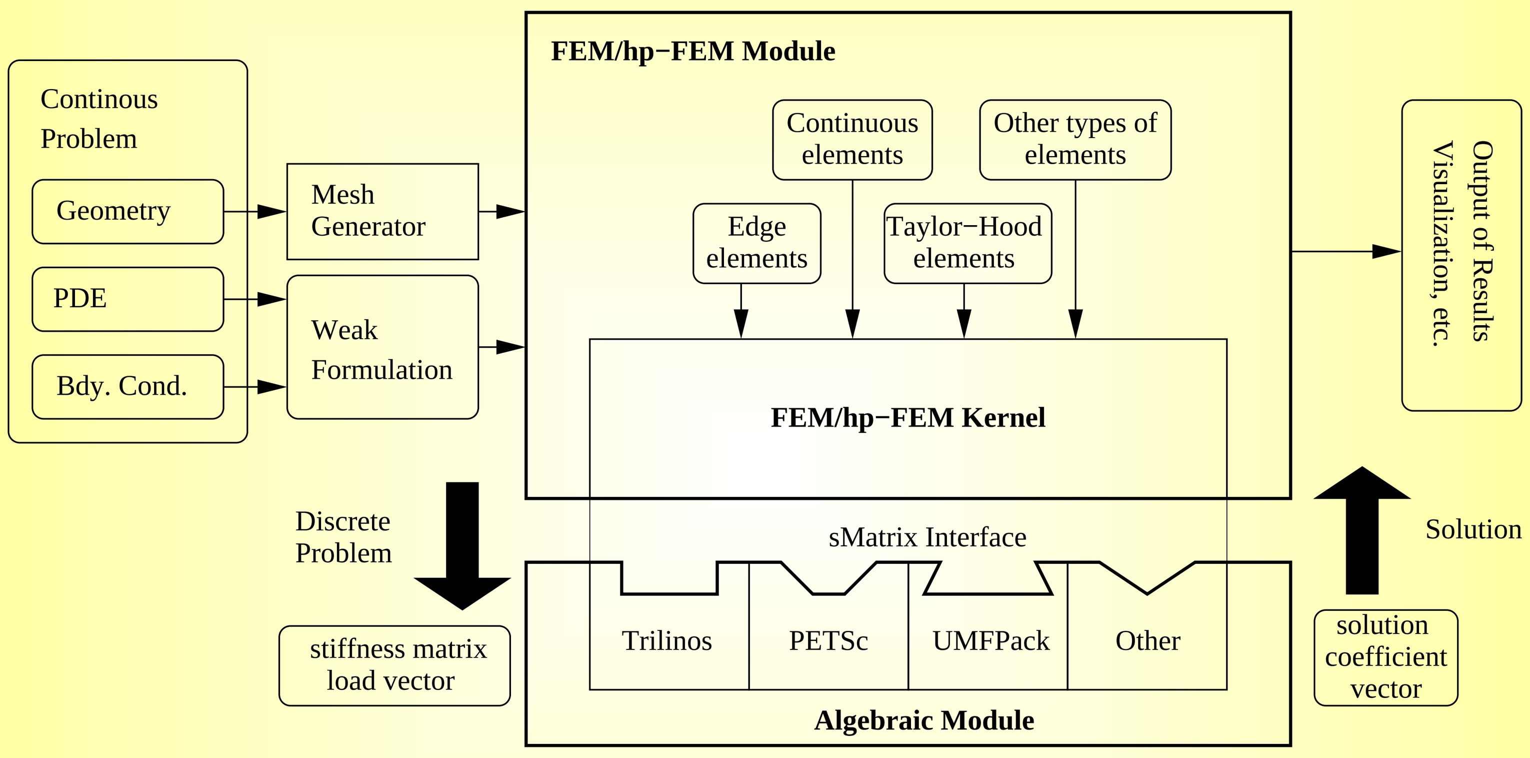
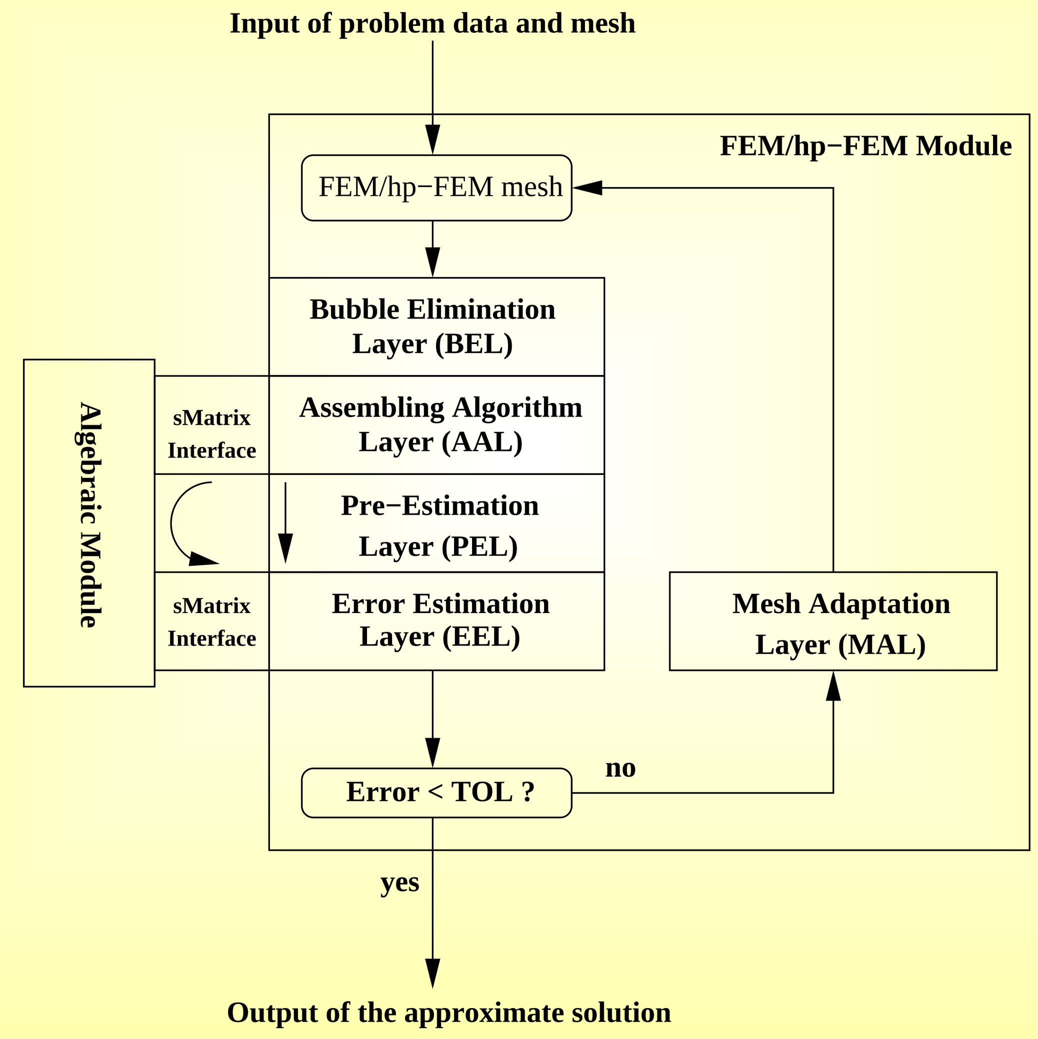
ABSTRACT: Partial differential equations (PDEs) describe a large variaty of natural processes, such as the weather, mechanics of solids, electromagnetics, fluid dynamics, heat transfer, chemistry, and others. Their efficient and accurate computer solution is extremely important for many engineering and scientific purposes. The best known method for the numerical solution of PDEs is the Finite Element Method (FEM). Its most sophisticated and efficient version is the hp-FEM, which is capable of achieving extremely fast (exponential) rates of convergence through simultaneous adaptive variation of the size h and polynomial degree p of the elements. We present the modular object-oriented finite element system HERMES, developed by the UTEPs Finite Element Group (http://servac.math.utep.edu/fem_group). The high efficiency of the hp-FEM comes at a price: Its computer implementation is substantially more demanding compared to standard FEM. Therefore it is essential to keep hp-FEM codes highly modular. We give a basic description of HERMES and present two illustrative examples, where the superiority of the hp-FEM over standard FEM (used in most engineering and commercial codes) is demonstrated.

Modular Structure of HERMES



- **FEM/hp-FEM Kernel:** The heart of HERMES: Collection of sophisticated PDE-independent FEM and hp-FEM algorithms, such as input/output, mesh processing and adaptation, general assembling and a-posteriori error estimation procedures, etc.
- **PDE Modules:** Small modules containing finite elements and data specific for various physical problems. Currently available are *continuous elements* (elasticity, electrostatics, stationary heat transfer) and *edge elements* (electromagnetics). *Taylor-Hood elements* (incompressible flow) are in progress.
- **Algebraic Module:** Collection of powerful iterative and direct algebraic solvers for systems of linear and nonlinear algebraic equations resulting from the FEM and hp-FEM discretizations.
- **sMatrix:** Universal interface that allows to switch among a variety of algebraic solvers, such as Trilinos, PETSc, or UMFPACK, without the need to alter the FEM/hp-FEM Module.

Algorithmic Flow Chart



Example 1: Electrostatic Micromotor

Electrostatic micromotors are microdevices capable of transforming electric energy into motion. They do not contain coils which could be destroyed by electromagnetic bombs. The device consists of fixed and mobile charged electrodes. In the following simulation we solve the standard potential equation of electrostatics in a plane-symmetric setting.

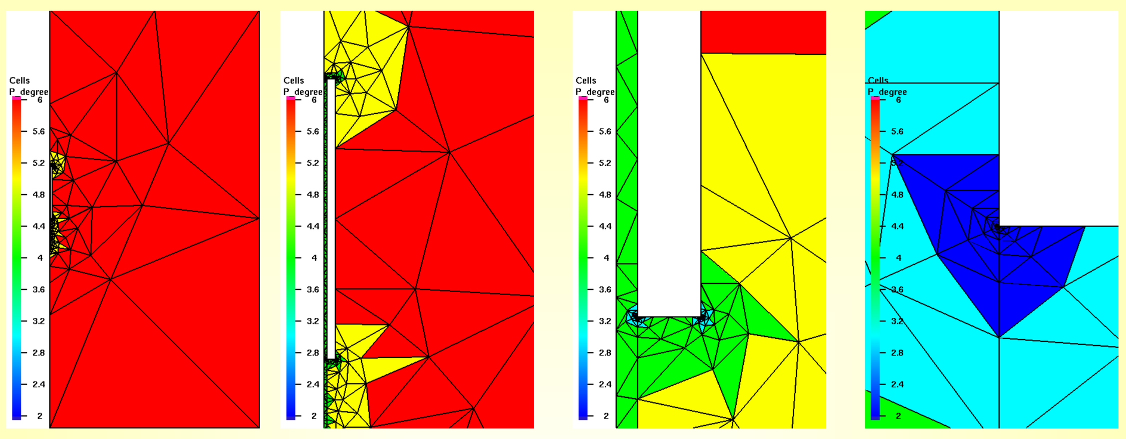


Figure 1: The hp-FEM mesh. Large sixth-degree elements are used far from the electrodes where the solution is "nice", and small quadratic elements are located at the corners of the electrodes in order to resolve the corner singularities. Zoom = 1, 6, 50, and 1000.

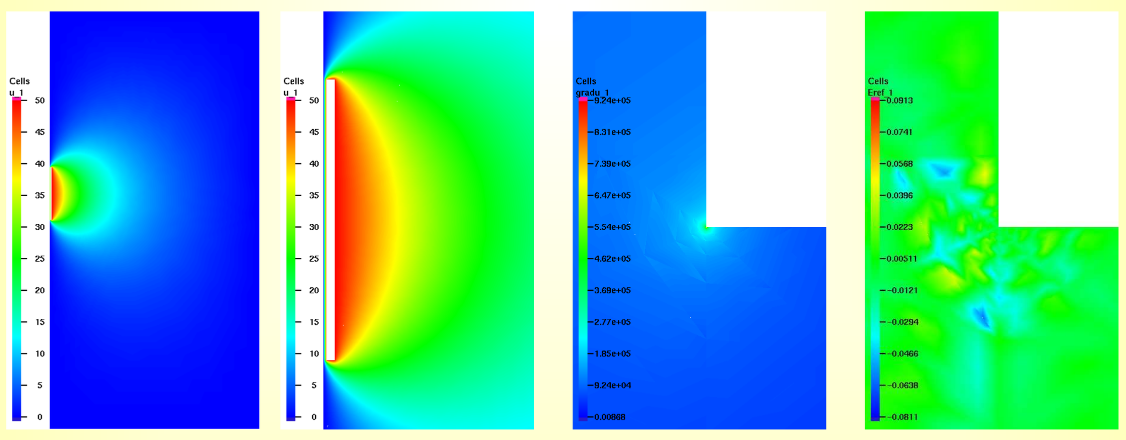


Figure 2: The approximate solution: (a) electric potential φ, (b) details of the fixed electrode (zoom = 6), (c) corner singularity of the electric field $\mathbf{E}$ (zoom = 1000), and (d) a-posteriori error estimate (zoom = 1000).

	Standard FEM	hp FEM
DOF	472384	4511
Error	0.2024%	0.173%
Iterations	387	71
CPU time	**32 min**	**17 sec**

Comparison of the number of DOF, relative error in the H^1-norm, number of iterations of the matrix solver, and the CPU-time.

Example 2: Electromagnetic Diffraction

The electromagnetic diffraction phenomenon occurs each time when an electromagnetic wave hits a corner or edge of a metallic object, such as an airplane. At such point the electric field develops a strong singularity which is easy to detect by a radar. Therefore, for example, the newest Stealth planes do not contain any sharp edges or corners on their surface. We solve a model problem formulated in terms of the time-harmonic Maxwell's equations, and use higher-order hierarchic edge elements.

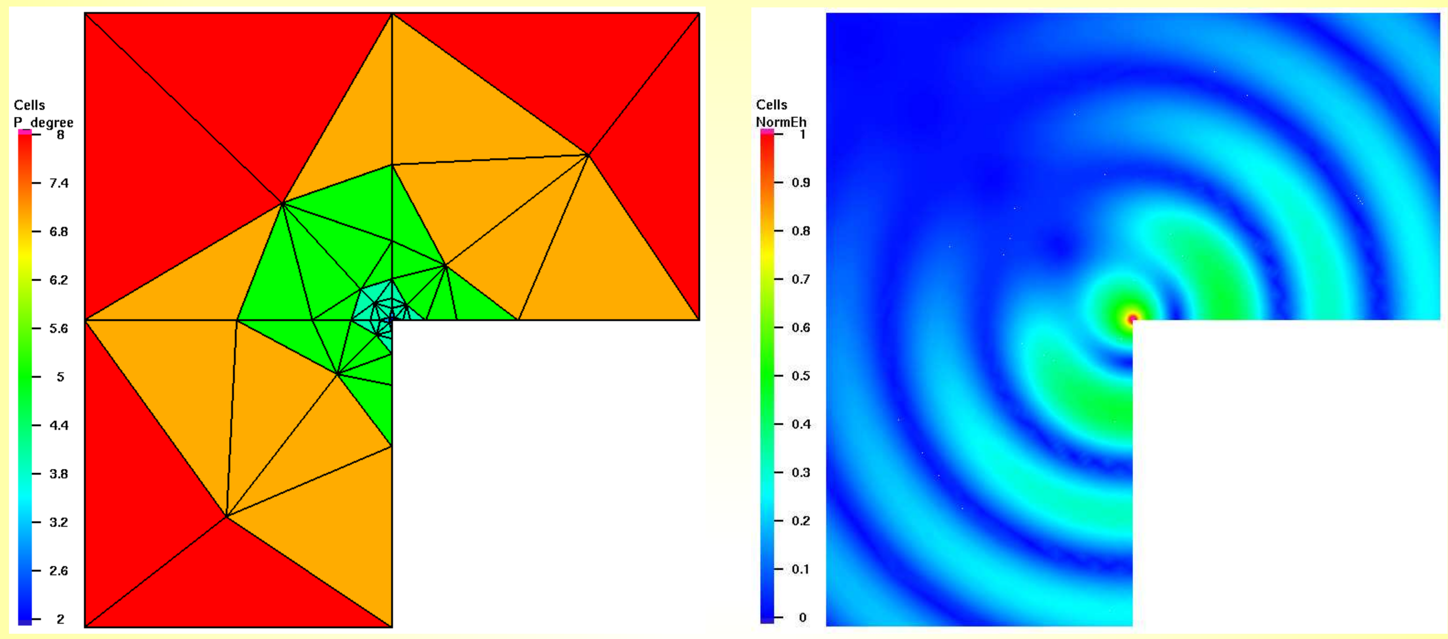


Figure 3: The hp-FEM mesh. Large eighth-degree elements are used far from the singularity and small quadratic elements are located at the reentrant corner.

	Standard FEM	hp FEM
DOF	2586540	4324
Error	0.6445%	0.6211%
CPU time	**21.2 min**	**2.49 sec**

Comparison of the number of DOF, relative error in the H^1-norm, and the CPU-time (this time, direct algebraic solver was used).

References

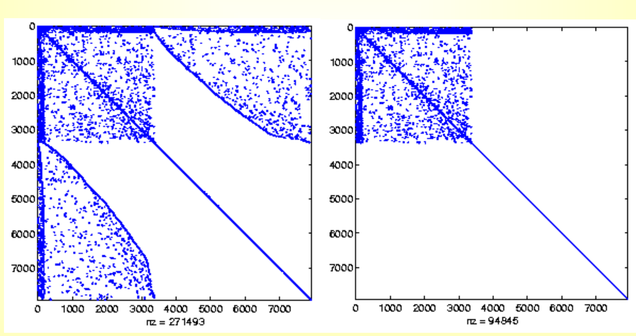
- More details on both computations and additional examples can be found in the book ***P. Šolín: Partial Differential Equations and the Finite Element Method, John Wiley & Sons, 2005.***
- A comprehensible discussion of the hp-FEM, including concrete data structures and algorithms used in HERMES, can be found in the monograph ***P. Šolín, K. Segeth, I. Doležel: Higher-Order Finite Element Methods, CRC Press/Chapmen & Hall, 2003.***
- For additional references, recent publications, and overview of current projects, please visit the home page of the UTEPs FEM Group **http://servac.math.utep.edu/fem_group**.

Computers

- Our central computing facility is a Cray XD1 Linux system ***Felina*** with 36 dual-core AMD Opteron processors @ 2.2 GHz, 144 GB RAM, and a peak performance of 317 GFlops. This computer was funded by the Department of Defense in the framework of the 2005 DURIP program.

Outlook

The hp-FEM contains a huge computational potential that has not been fully exploited yet. Open questions involve both the theory and algorithmization/implementation – the design of optimal higher-order elements for various PDEs capable of minimizing the size and condition number of the discrete problems, design of efficient scalable a-posteriori error estimation and automatic hp-adaptive procedures, parallelization and load balancing, etc.



This figure shows that with a suitable choice of higher-order shape functions, the size of the stiffness matrix may be reduced significantly. See our web page for details.

International Workshop FEMTEC 2006 at UTEP

The international conference ***Finite Element Methods in Science and Engineering*** took place at the University of Texas at El Paso on December 11–14, 2006. The meeting covered cover a variety of topics related to high-performance scientific computing with finite element methods: Reliability of results, computation with uncertain data, interval computation, spectral FEM, hp-FEM, extended and enriched FEM, etc. Among confirmed invited speakers were

- I. Babuška (ICES, UT Austin),
- T. Belytschko (Northwestern University),
- P. Bochev (Sandia NL),
- R. Glowinski (University of Houston),
- G. Karniadakis (Brown University),
- Rafi Muhanna (Virginia Tech).

For more information, please visit **http://www.math.utep.edu/femtec_2006/**.