Space-time adaptive two-mesh hp-FEM for an electromagnetic-thermal coupled problem

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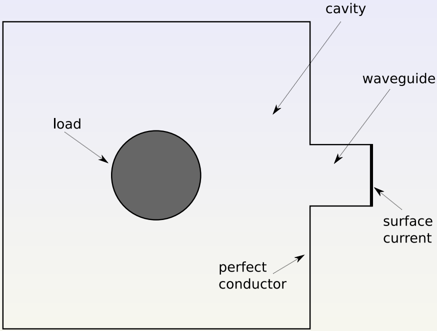
Outline

Microwave oven



- Electric field is generated by time-harmonic current.
- Frequency is chosen to be $f = 2.450$ GHz.
- Rounded load with different dielectric properties.
- Load is heated by the electric field energy.
- ϵ and γ depend on temperature.

$\Rightarrow$ Both ways coupling

Microwave heating - both ways coupling

Maxwell's Equations

$$\nabla \times (\mu_r^{-1} \nabla \times \boldsymbol{E}) - i\gamma(T)\kappa\sqrt{\frac{\mu_0}{\epsilon_0}}\boldsymbol{E} - \epsilon_r(T)\kappa^2 \boldsymbol{E} = 0 \text{ in } \Omega,$$

with boundary conditions

$$\begin{aligned} \boldsymbol{n} \times \left(\mu_r^{-1} \nabla \times \boldsymbol{E}\right) &= j\omega \boldsymbol{J}_s \quad \text{on waveguide end} \\ \boldsymbol{E} \cdot \boldsymbol{t} &= 0 \quad \text{elsewhere.} \end{aligned}$$

Heat-transfer equation

$$C_p \, \rho \, \frac{\partial T}{\partial t} - k \, \Delta T = \frac{1}{2}\gamma(T)|\boldsymbol{E}|^2$$

with boundary condition: $\frac{\partial T}{\partial n} = h(T - T_0)$, and initial condition: $T_0 = 20\ ^\circ C$

Linearization

On time level t_n we approximate

$$u \approx 2u^{n-1} - u^{n-2}$$

where u^{n-1} and u^{n-2} are solutions on previous time levels.

Maxwell's Equations

$$\nabla \times (\mu_r^{-1} \nabla \times \boldsymbol{E}) - i\gamma(2T^{n-1} - T^{n-2})\kappa\sqrt{\frac{\mu_0}{\epsilon_0}}\boldsymbol{E} - \epsilon_r(2T^{n-1} - T^{n-2})\kappa^2\boldsymbol{E} = 0$$

Heat-transfer equation

$$C_p\, \rho\, \frac{\partial T}{\partial t} - k\, \Delta T = \frac{1}{2}\gamma(2T^{n-1} - T^{n-2})|2\boldsymbol{E}^{n-1} - \boldsymbol{E}^{n-2}|^2$$

Rothe's method

We approximate the time derivative

$$\frac{\partial T}{\partial t} \approx \frac{T - T^{n-1}}{\tau}$$

where τ is the time step.

Maxwell's Equations

$$\nabla \times (\mu_r^{-1} \nabla \times \boldsymbol{E}) - i\gamma(2T^{n-1} - T^{n-2})\kappa\sqrt{\frac{\mu_0}{\epsilon_0}}\boldsymbol{E} - \epsilon_r(2T^{n-1} - T^{n-2})\kappa^2 \boldsymbol{E} = 0$$

Heat-transfer equation

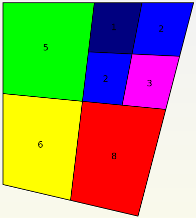
$$C_p \ \rho \ \frac{T}{\tau} - k \ \Delta T = C_p \ \rho \ \frac{T^{n-1}}{\tau} + \frac{1}{2}\gamma(2T^{n-1} - T^{n-2})|2\boldsymbol{E}^{n-1} - \boldsymbol{E}^{n-2}|^2$$

$\Rightarrow$ Time-independent linear equations.

Higher-order finite element method (space discretization)

hp-FEM

- Weak formulation.
- Galerkin approximation method.
- Partition of the domain ⇒ mesh (triangular, quadrilateral, mixed, curvilinear elements, irregular).
- *hp*-FEM allows different sizes (h) of elements and different polynomial orders (p) on elements.
- *hp*-FEM is capable of exponential convergence.

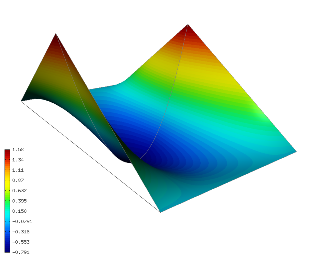


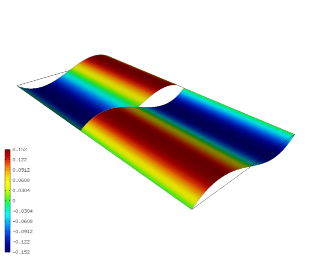
H(curl) space - basis functions (Electric field)

Conformity requirements:

- continuity inside each element K.
- continuity of tangential component across the edges.

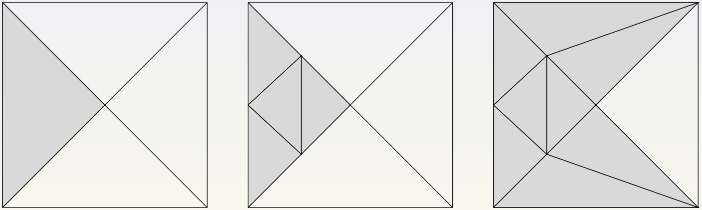
Basis functions:





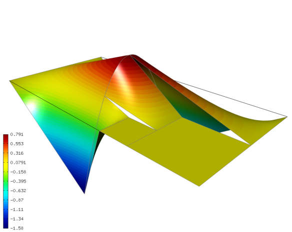
Why hanging nodes?

- Mesh refinement

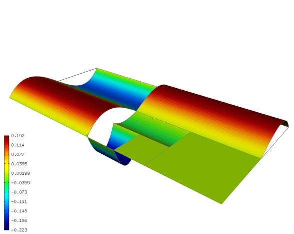


Higher level of hanging nodes

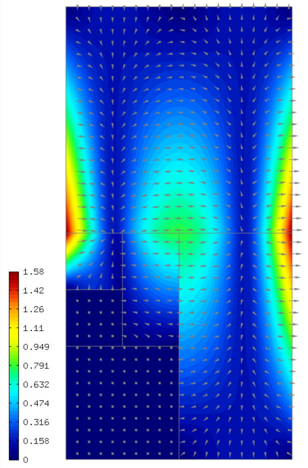
X-component



y-component



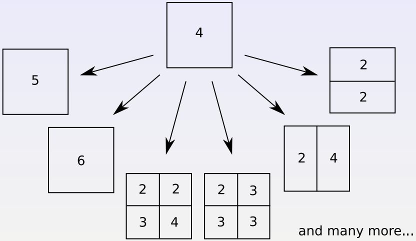
vector field (magnitude)



Adaptivity in *hp*-FEM

- Standard error estimaters are not sufficient.
- Many options how to refine an element.
- Information about shape of the error needed.

⇒ Reference solution



- Adaptivity on meshes with arbitrary-level hanging nodes is local.
- Elements adjacent to refined element are not affected.
- Forced refinements avoided.

⇒ Error is reduced optimally

Automatic adaptivity - algorithm

1. calculate the solution on the current mesh
2. calculate reference solution on refined mesh
3. calculate the error on each element and total error
4. sort all elements by their error
5. find the best refinements:
   - many candidates
   - project the reference solution onto these candidates
   - compare these projected solutions by their error and find the best candidate
6. recalculate the solution

Multi-mesh assembling

- Each field belongs to a different space - $\boldsymbol{H}(\text{curl})$ or H^1

  $\Rightarrow$ different finite elements for different components of the solution

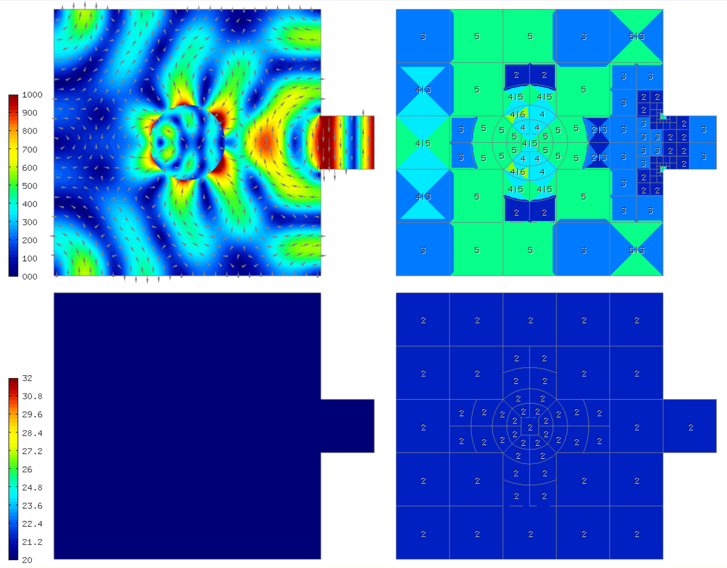
- Each field exhibits different behavior - singularities, boundary layers,...

  $\Rightarrow$ different meshes for different components of the solution

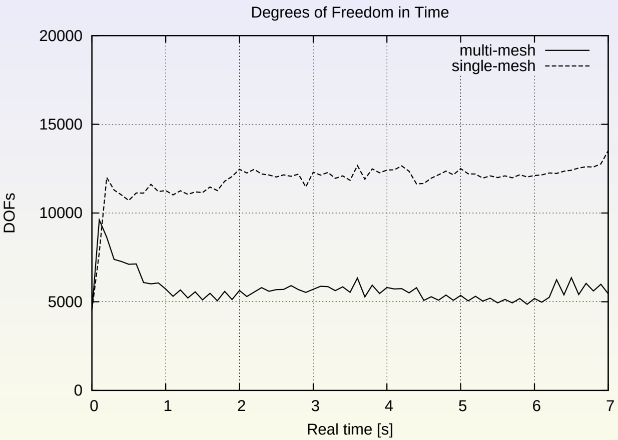
- The solution is changing in time

  $\Rightarrow$ different meshes for different time levels

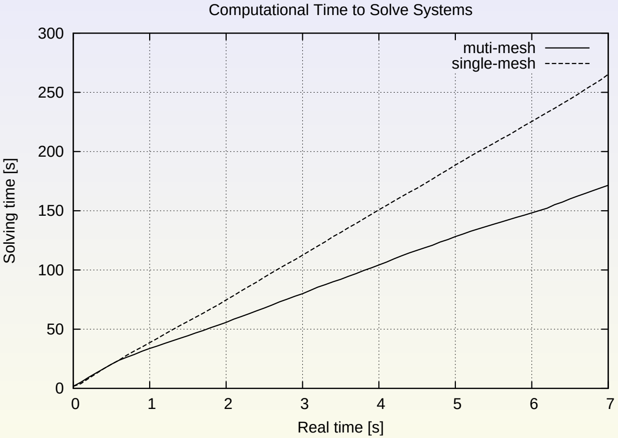
Electromagnetic–thermal problem



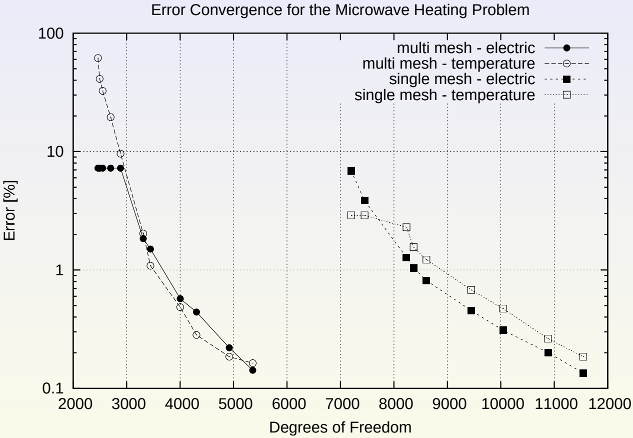
Comparison with single mesh - DOFs



Comparison with single mesh - solving time

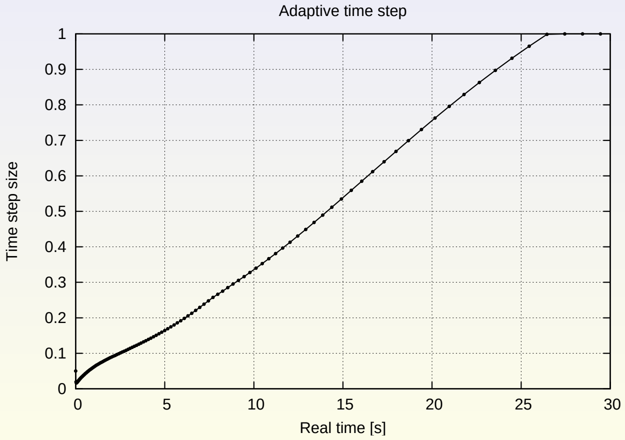


Convergence on time level $t = 5s$



Adaptive time step

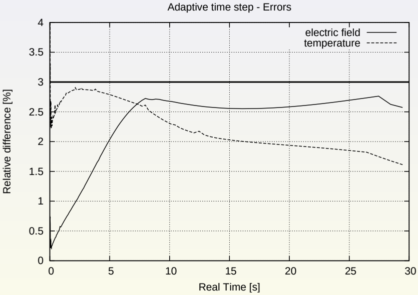
- Compare 2 successive solutions:
  - time step is too large $\Rightarrow$ use smaller τ and recalculate solution
  - time step is OK $\Rightarrow$ find new τ using PID controller



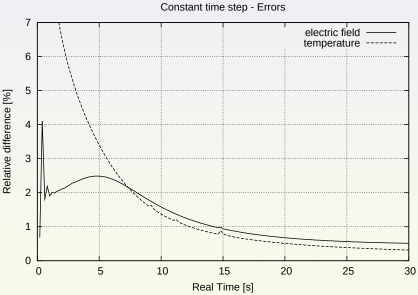
Adaptive time step

Difference between 2 successive solutions:

Adaptive time step



Constant time step, $\tau = 0.2$s



Conclusion

- Adaptivity in space and time.
- Irregular meshes.
- Different meshes for components.
- Different meshes in time.

Future work:

- Faster adaptivity.
- Other coupled problems - magnetohydrodynamics.

Thank you for your attention